**Distances of Superior Planets from the Sun**

Johannes Kepler was able to find the distance of the superior planets (Mars, Jupiter, and Saturn) from the Sun by using geometric drawings. That is, he made a scale drawing of the orbits of Earth and superior planets in which he called the distance between Earth and Sun 1 Astronomical Unit (A.U.).



From a study of the synodic periods the superior planets, Kepler was able to determine their sidereal periods. (See **Synodic and Sidereal Periods of the Planets**.)Once this was known for each planet, he would be able to determine the average daily motions of these planets. (Recall that Earth’s average daily motion is 360º/365.25d = 0.9856º/d.) Knowing how many degrees Earth had moved along in its orbit since the time of opposition (angle E1SE2), how far the superior planet had moved since the time of opposition (angle P1SP2), determining the difference between them (θ = E1SE2-P1SP2) and measuring the angular distance of the superior planet from the sun (ε), he was able to construct a scale triangle from which he was able to determine the distance of the superior planet from the sun in terms of Astronomical Units. (See diagram.) We will duplicate Kepler’s work by following these steps but will use a bit of trigonometry to assist us in our work.

1. Using Copernicus’ relationship for synodic and sidereal periods for inner and outer planets , determine the sidereal periods of Mars, Jupiter, and Saturn in days given their synodic periods in days.

Planet Synodic Period (d) Sidereal Period (d) Average daily motion (º/d)

**Mars** 779.94

**Jupiter** 398.88

**Saturn** 378.09

1. Calculate the average daily motion of each superior planet; divide 360º by the sidereal period of each planet. Record that information in the correct place above.
2. Consider the recent oppositions of the superior planets as well as their elongations from the sun (ε) on the following dates:

Opposition date Elongation (ε) by date Days between dates

**Mars** March 3, 2012 92.11º on June 3, 2012

**Jupiter** Dec. 2, 2012 78.14ºon March 10, 2013

**Saturn** April 28, 2013 120.21º on June 25, 2013

1. First, multiply the days between dates for Earth by its average daily motion to find out how far it has moved along in its orbit since the time of opposition and the elongations given in step 3 above. Then, do the same for each of the planets.

Average daily motion of Earth x days between dates = E1SE2 =

Average daily motion of **Mars** x days between dates = P1SP2 =

Average daily motion of Earth x days between dates = E1SE2 =

Average daily motion of **Jupiter** x days between dates = P1SP2 =

Average daily motion of Earth x days between dates = E1SE2 =

Average daily motion of **Saturn** x days between dates = P1SP2 =

1. Determine the value of θ in the above diagram by taking the differences between E1SE2 and P1SP2 for each of the above pairs. That is, (θ = E1SE2-P1SP2).

 θ ε ϕ

**Mars**

**Jupiter**

**Saturn**

1. Next, find ϕ for each of the planets by adding θ + ε and subtracting this sum from 180º, the total number of degrees in a triangle. Record that data in the table above.
2. Using your knowledge of trigonometry, determine the distance of the planet from the Sun in AU using the following formula: 

Calculated distance (AU)

**Mars**

**Jupiter**

**Saturn**

1. Compare your values with the average modern values in the table below.

Average distance (AU)

**Mars** 1.52

**Jupiter** 5.20

**Saturn** 9.58

Note that these values doe not match precisely the values found in step 7. This is primarily because of an assumption (uniform circular motion) that is incorrect.